1a) $\bar{z} + (1+i)z = 2 + \frac{i}{2}$

Låt $z = a + ib$, $a, b \in \mathbb{R}$.

$\bar{z} = a - ib$

fyll i, övning

$\Rightarrow \quad a - ib + (1+i)(a+ib) = 2 + \frac{i}{2} \Leftrightarrow \ldots \Leftrightarrow \begin{cases} 2a - b = 2 \\ a = \frac{1}{2} \end{cases} \Leftrightarrow \begin{cases} a = 1/2 \\ b = -1 \end{cases}$

ger $z = \frac{1}{2} - i$

Svar: $z = \frac{1}{2} - i$

övning! OBS: vi skriver om ett uttryck! alltså "=" tecken gäller!

1b) $z = \dfrac{(2+2i)(1+i\sqrt{3})}{3i(\sqrt{12} - 2i)} = \ldots =$

$= \dfrac{\cancel{2}\cdot\sqrt{2}\cdot e^{\frac{\pi}{4}i}\cdot\cancel{2}\cdot e^{\frac{\pi}{3}i}}{3\cdot e^{\frac{\pi}{2}i}\cdot\cancel{2}\cdot\cancel{4}e^{-\frac{\pi}{6}i}} = \dfrac{\sqrt{2}}{3}e^{\left(\frac{\pi}{4}+\frac{\pi}{3}-\frac{\pi}{2}+\frac{\pi}{6}\right)i} = \dfrac{\sqrt{2}}{3}e^{\frac{\pi}{4}i}$

Svar: $z = \dfrac{\sqrt{2}}{3}e^{\frac{\pi}{4}i}$

2. $3x^4 + 6x^2 = 8x^3 + 2$, antal reella lösningar sökes.

Bilda $f(x) = 3x^4 + 6x^2 - 8x^3 - 2$

$f'(x) = \ldots = 12x(x^2 - 2x + 1) = 12x(x-1)^2$

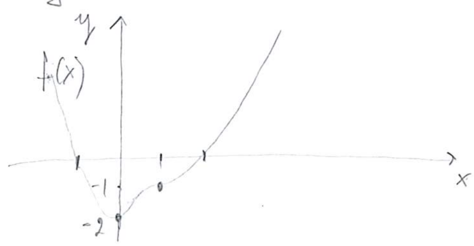
fyll i

$f'(x) = 0 \Rightarrow 12x = 0$ eller $x - 1 = 0 \Leftrightarrow x = 0$ eller $x = 1$

		0		1	
$f'(x)$	$-$	0	$+$	0	$+$
$f(x)$	$+\infty \searrow$	$f(0) = -2$	$\nearrow$	$f(1) = -1$	$\nearrow \rightarrow +\infty$

$\lim_{x \to \pm\infty} f(x) = \lim_{x \pm \infty} \underbrace{x^4}_{\to +\infty}\Big(3 + \underbrace{\frac{6}{x^2}}_{\to 0} - \underbrace{\frac{8}{x}}_{\to 0} - \underbrace{\frac{2}{x^4}}_{\to 0}\Big) = +\infty \cdot 3 = +\infty$

· f kontinuerlig



redovisning visar att $f(x) = 0$ precis 2 ggr.

Svar: Ekvationen har 2 stycken reella lösningar. Ser bilden med!

(2 komplexa)

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3. a) $\lim_{x\to 3} \frac{x^3-x^2-5x-3}{x^2-x-6} = \left[\text{typ} = "\frac{0}{0}" \right] \underset{\substack{\uparrow \\ \text{fyll i}}}{=} \ldots = \lim_{x\to 3} \frac{\cancel{(x-3)}^{1}(x^2+2x+1)}{\cancel{(x-3)}_{1}(x+2)} = \frac{16}{5}$

svar: $\frac{16}{5}$

b) $\lim_{x\to\infty} \left(\sqrt{x^2+2x} - \sqrt{x^2-3x}\right) \overset{\substack{\text{fyll i} \\ \downarrow}}{=} \ldots = \lim_{x\to\infty} \frac{x^2+2x-(x^2-3x)}{\sqrt{x^2+2x}+\sqrt{x^2-3x}} =$

$= \lim_{x\to\infty} \frac{5x}{|x|\cdot\sqrt{1+2/x} + |x|\cdot\sqrt{1-3/x}} = \left[x \to +\infty \Rightarrow |x| = x \right] =$

$= \lim_{x\to\infty} \frac{5\cancel{x}^{1}}{\cancel{x}_{1}\left(\sqrt{1+2/x}+\sqrt{1-3/x}\right)} = \lim_{x\to\infty} \frac{5}{\sqrt{1+\underbrace{\frac{2}{x}}_{\to 0}} + \sqrt{1-\underbrace{\frac{3}{x}}_{\to 0}}} = \frac{5}{1+1} = \frac{5}{2}$

svar: $\frac{5}{2}$

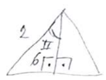
c) $\lim_{x\to 0} \frac{4x}{3\sin(2x)} = \lim_{x\to 0} \frac{4\cancel{x}^{1}}{3\cdot \underbrace{\frac{\sin(2x)}{2x}}_{\to 1 \text{ enl. of stand. gränsvärde}} \cdot \cancel{2x}_{1}} = \frac{\cancel{4}^{2}}{3\cdot 1\cdot \cancel{2}_{1}} = \frac{2}{3}$

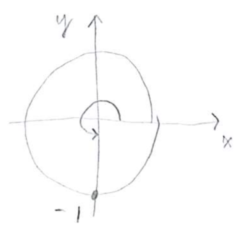
svar: $\frac{2}{3}$

4. a) $-2\cos^2 x + \sin x + 1 = 0 \iff$
$\iff -2(1-\sin^2 x) + \sin x + 1 = 0 \iff \left[\begin{array}{c} \sin x = t \\ \text{där} \\ -1 \leq t \leq 1 \end{array} \right] \iff 2t^2 + t - 1 = 0 \iff t^2 + \frac{1}{2}t - \frac{1}{2} = 0 \iff$

$\iff \left(t+\frac{1}{4}\right)^2 - \frac{9}{16} = 0 \iff \left(t+\frac{1}{4}+\frac{3}{4}\right)\left(t+\frac{1}{4}-\frac{3}{4}\right) = 0 \iff t+1=0 \text{ eller } t-\frac{1}{2}=0 \iff t=-1 \text{ eller } t=\frac{1}{2} \iff$

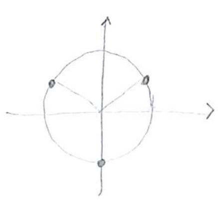
$\iff \left[t = \sin x \right] \iff \sin x = -1 \text{ eller } \sin x = \frac{1}{2} \iff x = \frac{3\pi}{2} + 2\pi n \text{ eller } \left\{ \begin{array}{l} x = \frac{\pi}{6} + 2\pi n \\ \text{eller} \\ x = \frac{5\pi}{6} + 2\pi n \end{array} \right.$, där $n \in \mathbb{Z} \iff$





$\iff x = \frac{\pi}{6} + \frac{2\pi}{3}n,$
$n \in \mathbb{Z}$

Alltså



lösningen ges av $x = \frac{\pi}{6} + \frac{2\pi}{3}n, \ n \in \mathbb{Z}$

svar: $x = \frac{\pi}{6} + \frac{2\pi}{3}n, \ n \in \mathbb{Z}$

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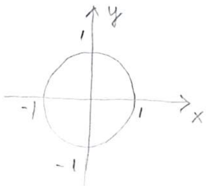
4b)

$\sin 2x = |\cos x|, \ \underline{0 \leq x \leq 2\pi}$



, $|\cos x| = \begin{cases} \cos x, & 0 \leq x \leq \frac{\pi}{2} \text{ eller } \frac{3\pi}{2} \leq x \leq 2\pi \\ -\cos x, & \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$

- fall 1

$\left(0 \leq x \leq \frac{\pi}{2} \text{ eller } \frac{3\pi}{2} \leq x \leq 2\pi\right)$ ger $\sin 2x = \cos x \Leftrightarrow 2\sin x \cos x = \cos x \Leftrightarrow$

$\Leftrightarrow 2\sin x \cos x - \cos x = 0 \Leftrightarrow \cos x(2\sin x - 1) = 0 \Leftrightarrow \cos x = 0 \text{ eller } 2\sin x - 1 = 0 \Leftrightarrow$

$\Leftrightarrow \cos x = 0 \text{ eller } \sin x = \frac{1}{2} \underset{\uparrow}{\Rightarrow} \left(x = \frac{\pi}{2} \text{ eller } x = \frac{3\pi}{2}\right) \text{ eller } x = \frac{\pi}{6}$

OBS!
för "$\Leftrightarrow$"
krävs samtliga lösningar
och vi
söker bara x för givna intervall

- fall 2

övning, fyll i

för $\frac{\pi}{2} < x < \frac{3\pi}{2}$ blir ekvationen $\sin 2x = -\cos x \Leftrightarrow \ldots \Leftrightarrow \cos x(2\sin x + 1) = 0$

ty $\cos x \neq 0$ för $\frac{\pi}{2} < x < \frac{3\pi}{2}$ sökes lösning för $\sin x = -\frac{1}{2}$ i given intervall,

Alltså $\sin x = -\frac{1}{2} \underset{\uparrow}{\Rightarrow} x = \frac{7\pi}{6}$

OBS!
tecken!

svar: $x = \frac{\pi}{2}, \ x = \frac{3\pi}{2}, \ x = \frac{\pi}{6}, \ x = \frac{7\pi}{6}$

4c) $\cos 2x - \sin 2x = 1, \ 0 \leq x \leq \pi$

$\cos 2x - \sin 2x = 1 \underset{\uparrow}{\Leftrightarrow} \ldots \Leftrightarrow \sin\left(\frac{\pi}{4} - 2x\right) = \frac{1}{\sqrt{2}} \Leftrightarrow \begin{cases} \frac{\pi}{4} - 2x = \frac{\pi}{4} + 2\pi n \\ \text{eller} \\ \frac{\pi}{4} - 2x = \frac{3\pi}{4} + 2\pi n \end{cases}, \ n \in \mathbb{Z} \Leftrightarrow \ldots \Leftrightarrow$

övning

$\Leftrightarrow \begin{cases} x = -\pi n \\ \text{eller} \\ x = -\frac{\pi}{4} - \pi n \end{cases}, \ n \in \mathbb{Z}$, lösningarna i intervallet $0 \leq x \leq \pi$ ges av ...

övning fyll i

svar: $x = 0$ eller $x = \frac{3\pi}{4}$ eller $x = \pi$

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5. $f(x) = \ln x + \arctan(1-x), \quad x > 0$

- $D_f: \ x > 0$
- $f'(x) = \frac{1}{x} + \frac{1}{1+(1-x)^2} \cdot (-1) = \underset{\text{fyll i övning}}{\dots} = \frac{(x-1)(x-2)}{x(x^2-2x+2)} = \frac{(x-1)(x-2)}{\underbrace{x}_{>0} \cdot \underbrace{((x-1)^2+1)}_{>0}}$

- $f'(x) = 0 \Rightarrow x-1=0$ eller $x-2=0 \Leftrightarrow x=1$ eller $x=2$

- OBS! derivatans nämnare >0 för alla $x>0$, alltså nämnarens tecken påverkar inte derivatans tecken

-

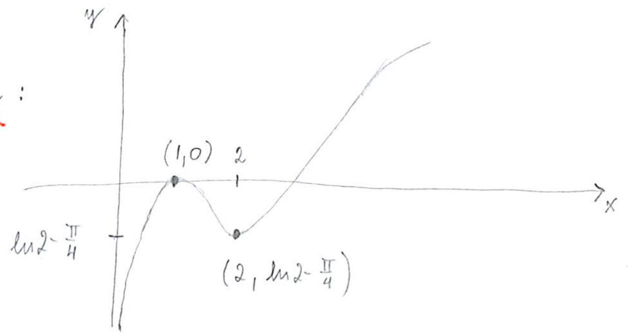
	0		1		2	
$f'(x)$		$+$	0	$-$	0	$+$
$f(x)$		$\nearrow$	lokal max $f(1)$	$\searrow$	lokal min $f(2)$	$\nearrow$

- $f(1) = \ln 1 + \arctan 0 = 0$
  $f(2) = \ln 2 + \arctan(-1) = \ln 2 - \frac{\pi}{4} < 0$

- $\lim_{x \to 0^+} (\ln x + \arctan(1-x)) = -\infty + \frac{\pi}{4} = -\infty$

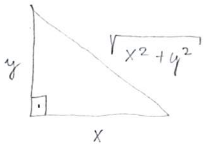
  $\lim_{x \to +\infty} (\ln x + \arctan(1-x)) = +\infty - \frac{\pi}{2} = +\infty$

Svar:



Lokalt max i $(1,0)$.
Lokalt min i $(2, \ln 2 - \frac{\pi}{4})$.

6.



Givet: $x + y + \sqrt{x^2+y^2} = 2 \iff$

$\iff \sqrt{x^2+y^2} = 2 - (x+y) \Rightarrow$ OBS! tecken

$\iff \cancel{x^2}+\cancel{y^2} = 2 - 4(x+y) + \cancel{x^2}+\cancel{y^2} + 2xy \iff$

OBS! tecken

$\iff 4y - 2xy = 4 - 4x \iff 2y - xy = 2 - 2x \iff y = \dfrac{2-2x}{2-x}$ där vi vet att $2-x>0$

ty även $y > 0 \Rightarrow 2-2x > 0 \iff x < 1$, alltså $\underline{\underline{0 < x < 1}}$

$A = \dfrac{x \cdot y}{2} = f(x) = \dfrac{\cancel{2}x(1-x)}{\cancel{2}(2-x)}$ ger $A = f(x) = \dfrac{x - x^2}{2-x}$ och $0 < x < 1$

- Sök nu $f_{max} = A_{max}$
- $f'(x) = \underset{\substack{\uparrow \\ \text{övning} \\ \text{fyll i}}}{\ldots} = \dfrac{x^2-4x+2}{(2-x)^2} = \dfrac{(x-2)^2-2}{(2-x)^2}$
- $f'(x) = 0 \Rightarrow (x-2)^2 - 2 = 0 \iff (x-2)^2 = 2 \iff x - 2 = \pm\sqrt{2} \iff x = 2 \pm \sqrt{2}$

ty $x = 2+\sqrt{2} > 1 \Rightarrow$ ej av intresse! ty $0 < x < 1$

Alltså bara $x = 2-\sqrt{2} < 1$ gäller att på!

-

	0		$2-\sqrt{2}$		1
$f'(x)$		+		−	
$f(x)$		↗	lokal max	↘	

$f_{max}(2-\sqrt{2})$

$A_{max} = f_{max}(2-\sqrt{2}) = \dfrac{(2-\sqrt{2}) - (2-\sqrt{2})^2}{2-(2-\sqrt{2})} = \underset{\substack{\uparrow \\ \text{övning}}}{\ldots} = (3-2\sqrt{2})$ a.e.

Svar: $A_{max} = (3-2\sqrt{2})$ a.e.

7. $f(x)=(\ln x)^x, \quad x>1$

1. Visa att $f(x)$ har en deriverbar invers f^{-1}
2. Ange $D_{f^{-1}}$
3. Beräkna $(f^{-1})'(1)$

1. $f(x)=(\ln x)^x = e^{x\ln(\ln x)}$

$$f'(x)=(\ln x)^x\left(\ln(\ln x)+\cancel{x}\cdot\frac{1}{\ln x}\cdot\frac{1}{\cancel{x}}\right)=\underbrace{(\ln x)^x}_{>0}\underbrace{\left(\ln(\ln x)+\frac{1}{\ln x}\right)}_{>0 \text{ (OBS! se nedan)}}>0 \text{ för } x>1$$

Vi ska nu studera: (visa att $g(x)$ är just >0)

$$h(x)=\ln(\ln x)+\frac{1}{\ln x}, \quad x>1$$

$$h'(x)=\frac{1}{\ln x}\cdot\frac{1}{x}-\frac{1}{(\ln x)^2}\cdot\frac{1}{x}=\frac{1}{x}\left(\frac{\ln x-1}{(\ln x)^2}\right)$$

$h'(x)=0 \Rightarrow \ln x-1=0 \Leftrightarrow \underline{x=e}$ (OBS! $x>1$)

	1		e	
$h'(x)$		$-$	0	$+$
$h(x)$		$\searrow$	lokal min $h_{min}(e)=1$	$\nearrow$

ty $h_{min}(e)=1>0 \Rightarrow h(x)>0$ för $x>1$ ($\Rightarrow f'(x)>0$)

$f'(x)>0$ för $x>1 \Rightarrow f$ har en deriverbar invers

2. $D_{f^{-1}}=V_f=\,]0,\infty[$ för att

- $\lim_{x\to 1} e^{x\overbrace{\ln(\ln x)}^{\to -\infty}} = \left[\begin{array}{l}\ln x\to 0 \text{ då } x\to 1\\ \Rightarrow \ln(\ln x)\to-\infty\\ e^{1\cdot(-\infty)}\to 0\end{array}\right]=0$

- $\lim_{x\to\infty}(\ln x)^x=\infty$

3. $(f^{-1})'(1)=\left[(\ln x)^x=1 \Leftrightarrow x=e\right]=\dfrac{1}{f'(e)}=\underset{\substack{\uparrow\\ \text{övning}}}{\ldots}=1$

<u>Svar</u>: Apropå "visa" se ovan (1). $D_{f^{-1}}=\,]0,\infty[$, $(f^{-1})'(1)=1$